Rotational Dynamics

Relation between physical quantities in linear motion and rotational motion.

Linear motion				Rotational motion				
Quantities	Symbol	Formula	Unit	Quantities	Symbol	Form.	Unit	Relation
1. Displacement	s/x	–	m	Angular displacement	θ	$\theta = s/r$	rad	$s = r.\theta$
2. mass	m	–	kg	moment of inertia	I	$I = mr^2$	kgm^2	$I = mr^2$
3. velocity	v	$v = s/t$	m/s	Angular velocity / frequency	ω	$\omega = \theta/t$	$rads^{-1}$	$v = \omega r$
4. Acceleration	a	$a = v/t$	ms^{-2}	angular acceleration	α	$\alpha = \omega/t$	$rad\ s^{-2}$	$a = \alpha . r$
5. Linear momentum.	p	$p = mv$	$kgms^{-1}$	angular momentum	J or L	$J = I\omega$	kgm^2s^{-1}	$J = r.p$
6. Force	F	$F = m.a$	$kgms^{-2}$ (N)	Torque	τ	$\tau = I.\alpha$	Nm	$\tau = r.F$
7. Work	W	$W = F.s$	Joule	Work	W	$W = \tau\theta$	Joule(J)	
8. Power	P	$P = \frac{W}{t}$	Watt	Power	P	$P = \frac{\tau.\theta}{t}$	Watt (W)	

Equations of linear motion

1. $v = u + at$
2. $s = ut + \frac{1}{2}at^2$
3. $v^2 - u^2 = 2as$
4. $KE = \frac{1}{2}mv^2$

Equations of rotational motion

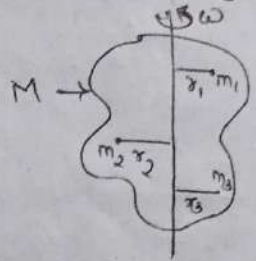
1. $\omega = \omega_0 + \alpha t$
2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
3. $\omega^2 - \omega_0^2 = 2\alpha\theta$
4. $KE = \frac{1}{2}I\omega^2$

Moment of inertia (I) :

It is physical quantity in rotational motion which play the same role as mass play in linear motion. Mathematically, I of a body about any axis of rotation is defined as the sum of product of mass and sq. of the ⊥r distance from of mass of particles from axis of rotation.

i.e. $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

$= \sum mr^2$; as shown in figure



It's unit is Kgm^2 in SI system.

Torque (τ):

It is a physical quantity in rotational motion, which play the same role as force play in linear motion.

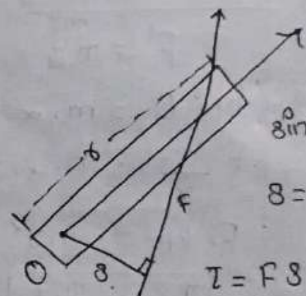
Mathematically, it is defined as the moment of force i.e. product of force and ⊥r distance of that force from the axis of rotation. It is denoted by τ and given by,

$\tau = r \cdot F$

In vector form,

$\vec{\tau} = \vec{r} \times \vec{F}$

It's unit is Nm in SI system.



$\sin\theta = \frac{S}{r}$

$S = r\sin\theta$

$\tau = FS$

$= Fr\sin\theta$

$\vec{\tau} = \vec{r} \times \vec{F}$

Relation between Torque and Moment of Inertia

$\boxed{\tau = I\alpha}$

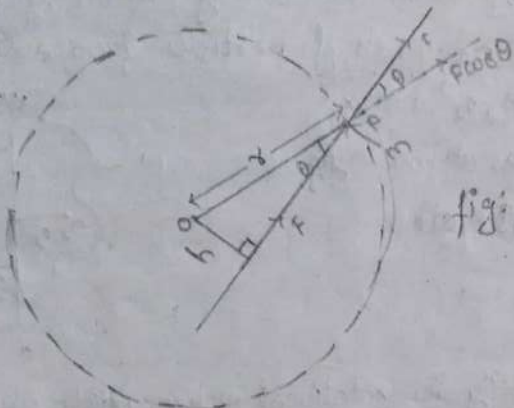


fig: rotation of mass

$$F' = ma$$
$$F\sin\theta = m.\alpha.r$$
$$F.\frac{h}{r} = m.\alpha.r$$
$$F.h = mr^2.\alpha$$
$$\boxed{\tau = I\alpha}$$

let us consider a massless rod of length OP whose one end 'O' is fixed and another end 'P' have mass 'm'. 'F' be the external force applied on the mass at point P by making angle 'θ' with axe. so that, there is two component of F, F cos θ & F sin θ.

The component, F sin θ = F' is responsible to bring the mass in linear motion with acceleration "a". Since, point 'O' is fixed. Then, rod is rotated about an axis passing through point 'O'.

Now, According to Newton's 2nd law of motion;

$$F' = ma$$

or, $F\sin\theta = m.\alpha r$; where 'α' is angular acceleration
'h' is ⊥r distance of Force (F) from axes of rotation.

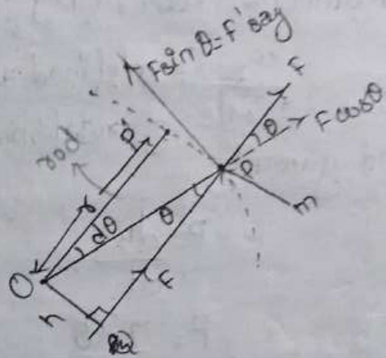
$\boxed{\tau = I\alpha}$ which is the required relation

Rotational Kinetic energy (E_{rot})

When a body is rotated due to torque, ($\tau = I\alpha$) —— ①
'I' is MI of body about axis of rotation and 'α' is angular acceleration.

Such that small angular displacement is '$d\theta$' then small workdone is given by,

$$dW = \tau\, d\theta$$



$$dW = I \cdot \alpha \cdot d\theta$$

$$dW = I \cdot \frac{d\omega}{dt} \cdot d\theta$$

$$dW = I \cdot \omega \cdot d\omega$$

Hence, total work done

$$W = \int_{\omega_i}^{\omega_f} I\, \omega\, d\omega$$

$$W = I \left[\frac{\omega^2}{2}\right]_{\omega_i}^{\omega_f}$$

$$W = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

$$\boxed{W = \text{Final K.E} - \text{Initial K.E.}}$$

In general, rotational KE $\boxed{(E_{rot}) = \frac{1}{2} I\omega^2}$

For a body revolving around the any fixed point or rolling body,

Total K.E. = $(K.E.)_{rot} + (K.E.)_{transitional}$

$$\boxed{E_T = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2.}$$

Put $v = \omega r$

$$E_T = \frac{1}{2} I\omega^2 + \frac{1}{2} m\omega^2 r^2$$

$$\boxed{E_T = \frac{1}{2} I\omega^2 + \frac{1}{2} m\omega^2 r^2}$$

Rotational Power (P):

It is defined as the rotational workdone per unit time, i.e. rate of rotational work done. It is denoted by 'P' and given by,

$$P = \frac{dW}{dt}$$

$$P = \frac{\tau d\theta}{dt}$$

$$\boxed{P = \tau\omega}$$

Oscillatory motion of spring mass system.

Angular Momentum and Conservation of Angular Momentum

Angular momentum is defined as the moment of linear momentum of a body. It is denoted by $\vec{L}$ and given by;

$$\vec{L} = \vec{r} \times \vec{P} \quad \text{---} \quad (1)$$

According to Newton's second law of motion, the external force acting on the body is defined as the rate of change of linear momentum. i.e.

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$\vec{r} \times \vec{F} = \cancel{\frac{d\vec{P}}{dt}} \; \vec{r} \times \frac{d\vec{P}}{dt}$$

$$\vec{\tau} = \vec{r} \times \frac{d\vec{P}}{dt} \quad \text{---} \quad (2)$$

Again,

$$\frac{d}{dt}(\vec{r} \times \vec{P}) = \vec{r} \times \frac{d\vec{P}}{dt} + \frac{d\vec{r}}{dt} \times \vec{P}$$

$$= \vec{r} \times \frac{d\vec{P}}{dt} + \cancel{\vec{v} \times m\vec{v}}^{\,0}$$

$$\therefore \frac{d}{dt}(\vec{r} \times \vec{P}) = \vec{r} \times \frac{d\vec{P}}{dt}$$

Eq^n (2) becomes

$$\vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{P}) \quad \text{---} \quad (3)$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \text{--- (4)}$$

Comparing (3) and (4)

$$\vec{L} = \vec{r} \times \vec{p}$$

proved.

Now, ~~conservation~~ principle of conservation of angular momentum states that "in absence of external torque, total angular momentum of a system always constant."

i.e. $\vec{L}$ = constant if $\vec{\tau} = 0$

Proof:

We know that,

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

If $\vec{\tau} = 0$ then,

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$d\vec{L} = 0$$

Integrating both sides,

$$\int d\vec{L} = \int 0$$

$$\Rightarrow \vec{L} = \text{const}$$

Since, $\vec{L} = I\vec{\omega}$

then,

$$\boxed{I_1\vec{\omega}_1 = I_1\vec{\omega}_2}$$

gr Numericals
Example: 8.1, 8.2, 8.4 — 104 Garcia
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rate of change of angular momentum.

Oscillatory Motion of Spring-Mass System.

The motion which repeats after certain interval of time is known as periodic motion. The periodic motion in a st. line is known as oscillatory motion. The periodic motion in which displacement is function of ~~similar~~ sine or cosine is known as harmonic motion. To and fro oscillatory harmonic motion in which acceleration is always directly proportional to the displacement and directed towards the mean position is known as simple harmonic motion. The motion of spring-mass system is simple harmonic.

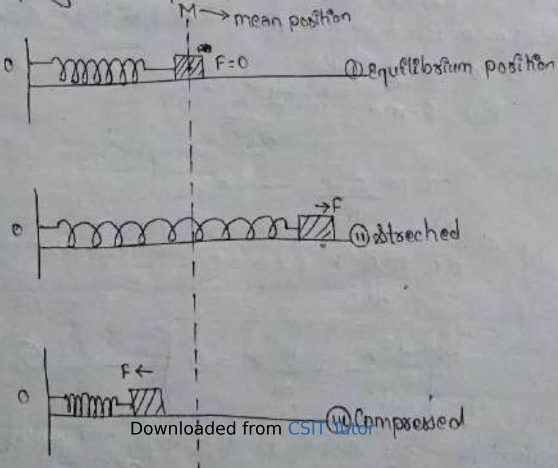


fig: Spring-mass system.

let us consider, a mass-~~having~~ less spring whose one end is fixed at a point O & another end is attached with a body of mass 'm' and placed in a horizontal frictionless surface, so that body is free to oscillate along that surface. In equilibrium position, ie in absence of external force, the body cannot oscillate as shown in fig(i).

If external force, F is applied on the body, so that spring streched in fig (ii) or compressed (fig (iii)). Restoring force (F_r) set up in the spring opposite to "F". When external force 'F' is removed then, body can oscillate about mean position.

If 'x' be the displacement of body of mass 'm'. then,

$$F_r \propto x$$

$$\text{or, } F_r = -Kx$$

$$\Rightarrow F = -Kx \text{ --- } (1)$$

where, K is proportionality constant known as spring constant or restoring force constant. (-)ve sign indicates that displacement is opposite to restoring force.

$$\text{Put } F = ma = m.\frac{d^2x}{dt^2}$$

$$\text{or, } m\frac{d^2x}{dt^2} = -Kx$$

$$\text{or, } \frac{m.d^2x}{dt^2} + Kx = 0$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + \frac{K}{m}x = 0} \text{ --- } (2)$$

which is the same as that of 2nd order differential equation of SHM; $\frac{d^2x}{dt^2} + \omega^2 x = 0$ --- (3)

where, 'ω' is the angular frequency of a body in SHM with displacement 'x'.

Equation ② and ③ shows that motion of spring mass system is simple harmonic motion (SHM).

Comparing eqn ② & ③;

$$\omega^2 = K/m$$

$$\Rightarrow \boxed{\omega = \sqrt{K/m}}$$

Characteristic of SHM of spring mass system

1. Frequency (f):

No. of oscillation per sec.

$$f = \frac{\omega}{2\pi}$$

$$\therefore f = \frac{1}{2\pi}\sqrt{K/m}$$

2. Time Period (T):

Time taken for one complete oscillation.

$$T = 1/f$$

$$T = 2\pi\sqrt{m/K}$$

3. Displacement (x):

Solution of ③ gives the displacement of particle (body) executing in SHM.

i.e. $x = A \sin \omega t$

where, A is maximum displacement also known as amplitude.

4. Velocity(v):

Rate of change of displacement.

i.e. $v = \frac{dx}{dt}$

$v = A\omega \cos\omega t$

$= A\omega\sqrt{1-\sin^2\omega t}$

$= \omega\sqrt{A^2 - A^2\sin^2\omega t}$

$\Rightarrow \boxed{v = \omega\sqrt{A^2 - x^2}}$

5. Acceleration (a):

Rate of change of velocity.

i.e. $a = \frac{dv}{dt}$

$= \frac{d(A\omega\cos\omega t)}{dt}$

$= A\omega \cdot -\omega\sin\omega t$

$= -A\omega^2\sin\omega t$

$\Rightarrow \boxed{a = -\omega^2 x}$

Energy of Harmonic Oscillation.

Total energy of particle executing in SHM is always equal to sum of K.E. and P.E.

i.e. $E = K.E. + P.E.$ - - - (1)

(1) Kinetic energy (K.E.):

let us consider a particle or body of mass 'm' is executing in SHM whose displacement at any instant of time 't' sec is given by;

P.T.O.

$x = A\sin\omega t$.

where, 'A' is maximum displacement also known as ~~magnitude~~ amplitude.

Now,

i.e. $v = \frac{dx}{dt}$

$\Rightarrow v = A\omega\cos\omega t = A\omega\sqrt{1-\sin^2\omega t} = \omega\sqrt{A^2 - A^2\sin^2\omega t}$.

$\Rightarrow \boxed{v = \omega\sqrt{A^2 - x^2}}$

We know,

$K.E. = \frac{1}{2}mv^2$

$$\boxed{K.E. = \frac{1}{2}m\omega^2(A^2 - x^2)} \quad \text{---(2)}$$

Potential Energy (P.E.):

When a particle executing in SHM is displaced from equilibrium position then there must be workdone against the restoring force which is stored is spring in form of P.E.

Small amount of workdone against the restoring force $F = -Kx$ for small displacement "dx" is

$dW = -F.dx$

or, $dW = Kx.dx$

$\therefore$ Total workdone $(W) = \int_0^x dW$

$= \int_0^x Kx\,dx$

$= K\left[\frac{x^2}{2}\right]_0^x$

$\therefore P.E = \frac{1}{2} K x^2$

Exi: 10, 2b, 2023
Num: 10.5, 10, 13b, 20, 18

Put $K = m\omega^2$;

$$\boxed{P.E = \frac{1}{2} m\omega^2 x^2} \quad \ldots (3)$$

Now,

Total energy $(E) = K.E. + P.E.$

$$= \frac{1}{2} m\omega^2 (A^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$

$$= \frac{1}{2} m\omega^2 A^2 - \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 x^2$$

$$\boxed{E = \frac{1}{2} m\omega^2 A^2 = \text{Constant.}} \quad 4$$

<u>Case-I</u>

At mean position; $x = 0$

$K.E = \frac{1}{2} m\omega^2 A^2 = E$ (maximum)

$P.E = 0$ (minimum)

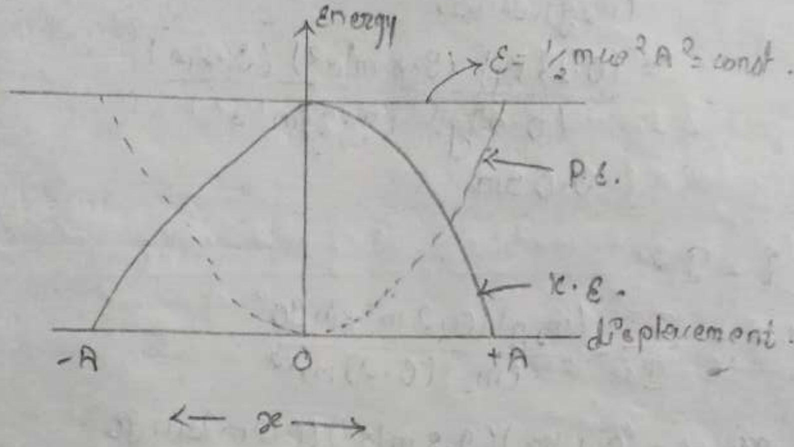
<u>Case-II</u>

At extreme position; $x = A$

$K.E = \frac{1}{2} m\omega^2 (A^2 - A^2) = 0$ (minimum)

$P.E = \frac{1}{2} m\omega^2 A^2 = E$ (maximum)

Hence, ~~variati~~ K.E, P.E. and total energy of particle ~~x~~ executing in SHM is varies with displacement (x) as shown in graph below;



Example: 8-1

A balance scale consisting of a weightless ~~pivot~~ rod has a mass of 0.1 kg on the right side 0.2 m from the pivot point. See fig 8.2(a) How far from the pivot point on the left must 0.4 kg be placed so that balance is achieved? (b) If the 0.4-kg mass is suddenly removed, what is the instantaneous rotational acceleration of the rod? (c) What is the instantaneous tangential acceleration of the 0.1 kg mass when 0.4 kg mass is removed?

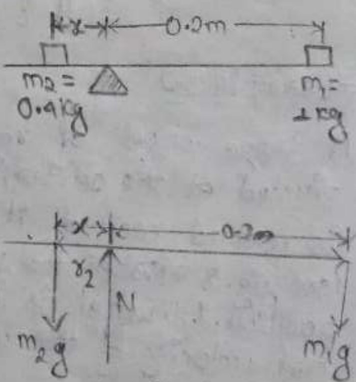


fig: 8-2.

a) When a balance is achieved $\alpha = 0$ &

$$\therefore \Sigma \tau = 0$$

On the right of the pivot the force is $m_1 g$ downward and the cross product $r \times F$ is into the paper or negative.
On the left the force is $m_2 g$ downward and the cross product $r \times F$ is out of the paper or positive.

$$(m_2 g)(x) \sin 90^\circ - (m_1 g)(0.2m) \sin 90^\circ = 0$$

Solving for x,

$$x = \frac{(m_1 g)(0.2m)\sin 90^\circ}{(m_2 g)\sin 90^\circ}$$

$$= \frac{(0.1 kg)(9.8 m/s^2)(0.2m)}{(0.4 kg)(9.8 m/s^2)}$$

$$= 0.05m$$

(b) $\tau = I\alpha$

or, $\alpha = \frac{\tau}{I} = \frac{(m_1 g)(0.2 m)\sin 90^\circ}{(m_1)(0.2 m)^2}$

or, $\alpha = \frac{(0.1 kg)(9.8 m/s^2)(0.2m)\sin 90^\circ}{(0.1 kg)(0.2 m^2)}$

$\therefore \alpha = 49 rad/s^2$ (clockwise)

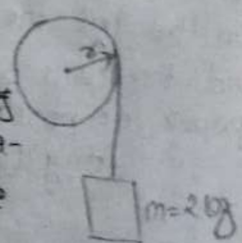
(c) $a_T = r\alpha$

$= (0.2m)(49 rad/s^2)$

$= 9.8 m/s^2$.

Example 8.2

A large wheel of radius 0.4m and moment of inertia 1.2 kgm^2, pivoted at the center, is free to ~~axle~~ rotate without friction. A rope is wound around it and a 2 kg weight is attached to the rope (see fig. 8.4) when the weight has descended 1.5m from its starting position. (a) what is its downward velocity? (b) what is the rotational velocity of the wheel?



→ Solution,

(a) We may solve this problem by the conservation of energy, equating the initial ~~energy~~ potential energy of the weight to it's conservation of kinetic energy of the weight and of the wheel.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

The downward velocity v of the weight is equal to the tangential velocity at the rim of the wheel v_r; therefore

$$\omega = \frac{v_r}{r} = \frac{v}{r}$$

Substituting for ω,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

We solve for velocity v;

$$v = \left[\frac{mgh}{\frac{1}{2}m + \frac{I}{2r^2}}\right]^{1/2}$$

$$= \left[\frac{(2\,kg)(9.8\,m\,s^{-2})(1.5\,m)}{\left(\frac{1}{2}\right)(2\,kg) + \frac{(1.2\,kg\,m^2)}{(2)(0.4\,m)^2}}\right]^{1/2}$$

$$v = 2.5\,m/sec$$

(b) The answer to part (a) shows that any point on the rim of wheel has a tangential velocity of $v_r = 2.5\,m/sec$. We convert this to rotational velocity of a wheel.

$$\omega = \frac{v_r}{r} = \frac{2.5\,m/sec}{0.4\,m} = 6.2\,rad/sec$$

Example 8.4

Suppose the body of an ice skater has a moment of inertia $I = 4\,kg\,m^2$ and her arms have a mass of 5 kg each with the center of mass at 0.4 m from be her body. She starts to turn at 0.5 rev/sec on the point of her skate with her arms outstretched. She then pulls her arms inwards so that their center of mass is at the axis of her body, $r = 0$. What with be her speed of rotation?

→ Solution,

$$I_0\omega_0 = I_f\omega_f$$
$$(I_{body} + I_{arms})\omega_0 = I_{body}\omega_f$$
$$(I_{body} + 2mr^2)\omega_0 = I_{body}\omega_f$$

Solving for ω_f

$$\omega_f = \frac{(I_{body} + 2mr^2)\omega_0}{I_{body}} = \frac{[4\,kg\cdot m^2 + 2\times 5\,kg\times(0.4\,m)^2](0.5\,rev/sec)}{4\,kg\,m^2}$$
$$= 0.7\ rev/sec.$$

Problems #8.1

A bicycle wheel of mass 2 kg and radius 0.32m is spinning freely on its axle at 2 rev/sec. When you place your hand against the tire the wheel decelerates uniformly and comes to stop in 8 sec. What is the torque of your hand against the wheel?

rad/s → ω
rev/s → f.

→ Sol^n,

Here; mass of wheel (m) = 2 kg
radius of wheel (r) = 0.32 m
frequency of the wheel (f_0) = 2 rev/sec
time taken to stop (t) = 8 sec
Initial angular velocity (ω_0) = $2\pi f_0$
$= 2\pi\times 2 = 4\pi$ rad/sec.

Final angular velocity (ω_2) = 0 {stops}
Torque of hand against wheel (τ) = ?

Now,

we have;
$$\omega_2 = \omega_0 + \alpha t$$
or, $0 = 4\pi + \alpha\cdot 8$
$$\therefore \alpha = -\pi/2\ rad/s^2 = 1.57\ rad/s^2$$

~~Torque (τ)~~

Inertia of wheel (I) = $m r^2$

$= 2\times(0.82)^2$

$= 1.3448\ kg\,m^2$

Now,

Torque(τ) = $I \cdot \alpha$

$= 1.3448 \times 1.57$

$= 2.111336\ Nm.$

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Problem 8.2

Two masses, $m_1 = 1$ kg and $m_2 = 5$ kg, are connected by a rigid rod of negligible weight (see fig 8.6). the system is pivoted about point (O). the gravitational forces act in the negative z direction. (a) Express the position vectors and the forces on the masses in terms of unit vectors and calculate the ~~torq~~ torque on the system. (b) What is the angular acceleration of the system at that instant shown in fig 8.6?

→ Soln,

Here; $\vec{r}_1 = \cancel{2m}\ -2\hat{j}\,m$

$\vec{r}_2 = 4\hat{j}\,m$

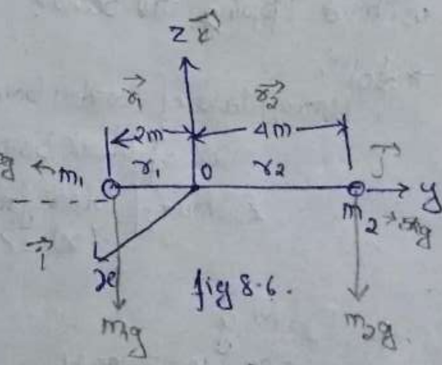


fig 8.6.

$\vec{F}_1 = -m_1 g\,\vec{k}\ N = -10\hat{k}\ N$

$\vec{F}_2 = -m_2 g\,\hat{k}\ N = -50\hat{k}\ N$

$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1 = -2\hat{j} \times -10\hat{k} = 20\hat{i}\ Nm$

$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_1 = 4\hat{j} \times -50\hat{k} = -200\hat{i}\ Nm$

(a) $\vec{r} = ?$; $\vec{F} = ?$; $\vec{\tau} = ?$

(b) $\alpha = ?$

$\vec{\tau} = \vec{\tau_1} + \vec{\tau_2}$
$= 20\hat{i} + (-200\hat{i}) = -180\hat{i}$ Nm

$I = \sum m r^2$
$= 1 \times (-2\hat{i})^2 + 5 \times (4\hat{i})^2$
$= 4 + 5 \times 16 = 80$ kgm^2

Also;
$\vec{\tau} = I \cdot \alpha$
$-180\hat{i} = 80 \cdot \alpha$
$\therefore \alpha = -10$ rad/s^2

Problem 8.7

A uniform wooden board of mass 20 kg rests on two supports as shown in fig 8-9. A 30 kg steel block is placed to the right of support A. How far to the right of A can the steel block be placed without tipping the board?

→ Soln,

Here; Mass of wooden board (M) = 20 kg
mass of steel board (m) = 20 kg
Distance of block from ~~board~~ A (x) = ?

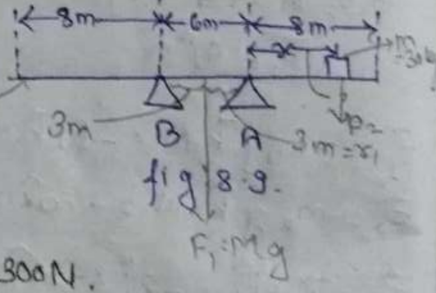


fig 8.9.

$F_1 = Mg$
$= 20 \times 10 = 200$ N

$F_2 = mg$
$= 30 \times 10 = 300$ N.

At equilibrium,
$r_1 F_1 = r_2 F_2$
~~200~~ $3 \times 200 = x \times 300$
$\therefore x = 2$ m

Unit-2 ELECTRIC & MAGNETIC FIELD

Electric field:

The space around the charge upto which it's effect can be observed is called electric field. If another charge is placed in the electric field, it experience force known as electrostatic force.

According to Columb, the electrostatic force bet^n any two charges q_1 & q_2, separated by distance 'r' in the medium of permittivity ϵ_0 is;

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

In vector form,

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r} = \frac{q_1 q_2 \vec{r}}{4\pi\epsilon_0 r^3} \left[\because \hat{r} = \frac{\vec{r}}{r}\right]$$

Permittivity (ϵ)

It is the property of medium, ~~by virtue of which~~ which gives the response to electrostatic force bet^n the charges when they are placed at that medium.

For example;

Permittivity of water is 80 times greater than that of air due to which electrostatic force between any two charges in air is 80 times greater than that in water when charges are separated by same distance.

Value of permittivity of air $= 8.85 \times 10^{-12}\, Fm^{-1}$;

↳ denoted by ϵ_0.

Relative permittivity (ϵ_r):

The ratio of permittivity of medium to the permittivity of air, is known as relative permittivity, denoted by ϵ_r and given by;

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

It is unit less and it's value is 1 for air.

Electric field Intensity (E)

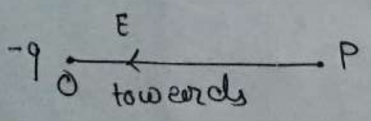
Electric field intensity at any point in the electric field is defined as the electrostatic force experience by unit +ve charge (+1C) placed at that point.

It is denoted by "E" and given by:

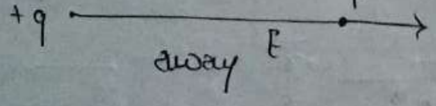
$$E = \frac{F}{q_0} \quad \ldots (1)$$

where, "F" is total force experience by +ve test charge "q_0".

It's unit is N/C or ~~N/m~~ V/m & is vector quantity. It is always directed towards -ve charge and away from +ve charge as shown in figure below



and



Expression for electric field intensity due to point charge.

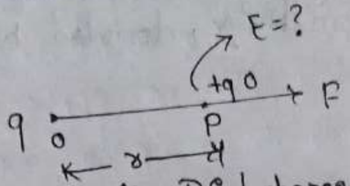


fig: Point charge

Let us make 'q' amount of charge at point 'O' known as point charge. 'P' is any point in the electric field at distance 'r' from point 'O'. To find electric field intensity 'E' at point 'P' let us place +ve test charge '+q' at point 'P'.

Here, force experience by given test charge

$$F = \frac{q \cdot q_0}{4\pi \epsilon_0 r^2} \quad \text{----} (1)$$

According to definition,

$$E = \frac{F}{q_0} \quad \text{----} (2)$$

From eqn (1) & (2)

$$E = \frac{q}{4\pi \epsilon_0 r^2}$$

In vector form,

$$\vec{E} = \frac{q}{4\pi \epsilon_0 r^2} \cdot \hat{r}$$

Put $\hat{r} = \frac{\vec{r}}{r}$, $\vec{E} = \frac{q}{4\pi \epsilon_0 r^3} \cdot \vec{r}$

For multiple charges $q_1 q_2, \ldots$

$\vec{E} = \vec{E_1} + \vec{E_2} + \ldots$

i.e. vector sum

~~for multiple charges q~~

Electric Potential (V)

Electric Potential at any point in the electric field is defined as the amount of work done against electrostatic force in moving unit +ve charge (+1C) from infinity to that point. It is denoted by 'V' & given by $V = W/q$ ---(1)

where $W \rightarrow$ total workdone in moving 'q' amount of charge. It's unit is J/C known as volt (V).

Electric Potential Difference ($V = V_{AB}$)

Electric Potential difference between point in the electric field is defined as the amount of work done against electrostatic force in moving unit +ve charge +1C from one point to another point. It is denoted by ($V = V_{AB}$) & given by;

$$V_{AB} = \frac{W_{AB}}{q} \quad ---(1)$$

where 'W' is total workdone in moving 'q' amount of charge from B to A. It's unit is J/C known as volt (V).

Expression for electric potential due to point charge

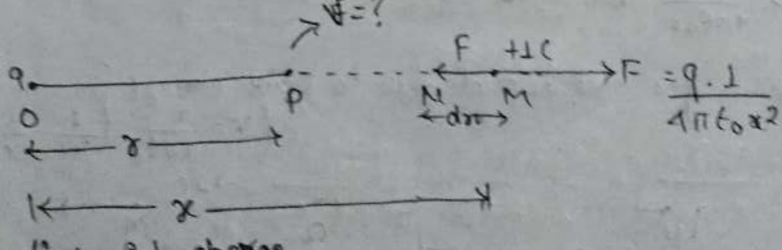


fig: point charge

Let us take 'q' amount of charge at point 'O' known as point charge. 'P' is any point at distance "r" from point 'O' to find electric potential (V) at point 'P', let us produce OP and placed unit +ve charge (+1C) at point 'M' at distance 'x' from point 'O'.

Force experience by given unit charge is

$$F = \frac{q_1 \cdot 1}{4\pi\epsilon_0 x^2} \quad \text{--- ①}$$

small amount of work done against this force to move given unit charge by small distance $dx = MN$ is $dW_x = -F\,dx$ --- ②

-ve sign indicate that force & displacement are opposite. From eqn ① & ②;

$$dW = \frac{-q}{4\pi\epsilon_0 x^2}\,dx \quad \text{--- ③}$$

Now, total work done in moving given unit charge from infinity [$x = \infty$] to point $P(x = r)$ is

$$W = \int_\infty^r dW = \int_\infty^r -F\,dx$$

$$= \int_\infty^r -\frac{q}{4\pi\epsilon_0 x^2}\,dx$$

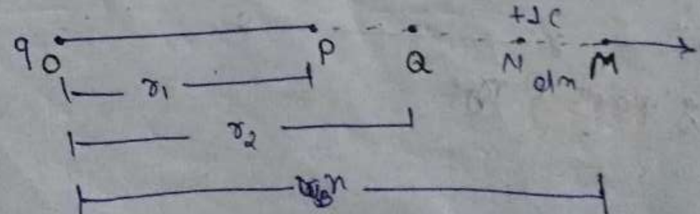
$$= \frac{-q}{4\pi\epsilon_0}\left[\frac{x^{-1}}{-1}\right]_\infty^r$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x}\right]_\infty^r$$

$V = W = \dfrac{q}{4\pi\epsilon_0 r}$ which is required expression.

Expression for electric p.d. due to point charge

$$\left(V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right)$$



Let us take 'q' amount of charge at point 'O' known as point charge. P and Q are any two points at a distance r_1 and r_2 respectively from the point 'O' to find the electric potential difference (V_{PQ}) between the points P and Q, let us produce OQ and place a unit +ve charge (+1C) at point M at the distance x from point 'O'.

Force experience by given unit charge;

$$F = \frac{q.1}{4\pi\epsilon_0 x^2} \quad --- (1)$$

Small amount of work done against this force to move given unit charge by small distance $dx = MN$ is

$$dW = -Fdx \quad --- (2)$$

-ve sign indicates that the force & displacement are opposite.

From eqn (1) & (2);

$$dW = \frac{-q}{4\pi\epsilon_0 x^2} . dx \quad --- (3)$$

Now, total work done in moving given unit charge from Q to $[x = r_2]$ to P $[x = r_1]$ is;

$$W = \int_{r_2}^{r_1} dW = \int_{r_2}^{r_1} \frac{-q}{4\pi\epsilon_0 x^2} dx$$

$$= \frac{-q}{4\pi\epsilon_0} \left[-\frac{1}{x} \right]_{r_2}^{r_1}$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$V_{PQ} = W = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$ which is required expression.

Electric Potential Energy (U)

Electric potential energy at any point in the electric field is defined as the amount of workdone in moving given amount of charge (q columb) from infinity to that point. It is denoted by 'U' and given by

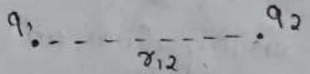
$$U = qV$$

It's unit is Joule (J)

For a system of two charges q_1 & q_2 separated by distance r_{12}

$$U_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

as shown in fig (1) below.



fig(1)

Again, for system of three charges as shown in fig(2) below

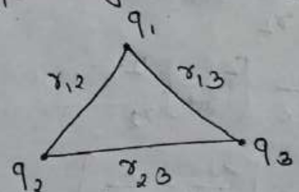


fig (2)

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} + \frac{q_1 q_3}{4\pi\epsilon_0 r_{13}} + \frac{q_2 q_3}{4\pi\epsilon_0 r_{23}}$$

For system of 'n' no. of charges $q_1, q_2, \ldots, q_N$.

$$U = \frac{1}{4\pi\epsilon_0} \sum_{\substack{i \\ j > i,\ j=1}}^{N} \frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0} \sum_{i,j=1}^{N}{}' \frac{q_i q_j}{r_{ij}}$$

One electron volt (1eV) Energy:

The kinetic energy gained by an electron when it is accelerated by the potential of 1 volt. ~~It's~~ If 'q' amount of charge is accelerated by potential of V volt then,

$$\text{gain in K.E} = \text{Work done}$$
$$\text{gain in K.E.} = qV \quad \text{----} (1)$$

If $q = e = 1.6 \times 10^{-19}\,C$

& $V = 1\text{ volt} = 1\,V$

then,

$$K.E. = 1\,eV$$

$\therefore$ eq^n (1) becomes; $1\,eV = 1.6 \times 10^{-19}\,C \times 1\,V$

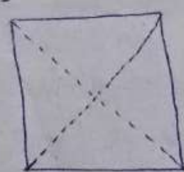
$$\boxed{1\,eV = 1.6 \times 10^{-19}\,J}$$

where, $CV = J$.

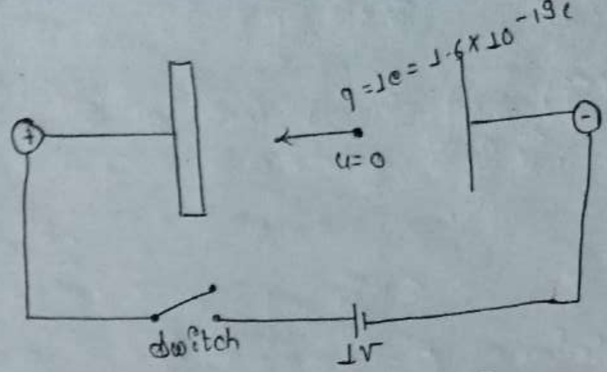
Example: 14.1, 14.2, 14.3
Problem: 14.6, 14.8 & 14.21

Problem 14.6

$q_1 =$



One electron volt (1eV) Energy:



Here, electron moves towards ⊕ when 1V is supplied with gain a certain K.E. i.e.,

$$K.E. = W = F.s = ma.s = m\frac{v^2}{2} = \frac{1}{2}mv^2$$

$\hookrightarrow qV = KE$

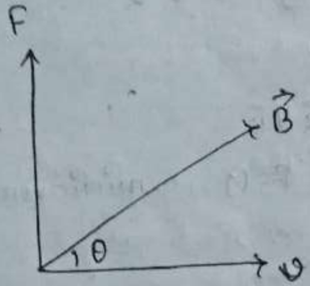
$1e \times 1$ volt

$1.6 \times 10^{-19} CV$

$1.6 \times 10^{-19} J = 1eV$

Magnetic Field

Force on a moving charge in uniform magnetic field (Lorentz force)



Force on moving charge in uniform m.f.

Let us consider a charge 'q' is moving with velocity '$\vec{v}$' in uniform motion field of intensity '$\vec{B}$' by making angle θ between '$\vec{v}$' as shown in fig {above}.

If '$\vec{F}$' be the force experienced by that charge then experimentally it is found that,

i) $F \propto B$

ii) $F \propto q$

iii) $F \propto v$

iv) $F \propto \sin\theta$

Combining above relation, we get,

$$F \propto Bqv\sin\theta$$

$$F = Bqv\sin\theta \quad \text{----(i)}$$

where, proportionality constant is taken as unity.

In vector form, eqⁿ ① can be written as,

$$\vec{F} = q(\vec{B} \times \vec{v}) \quad \text{---} (2)$$

Eqⁿ (2) shows that Lorentz force ($\vec{F}$) is always perpendicular to the plane containing $\vec{B}$ & $\vec{v}$.

Case I:

when $\theta = 0°$; i.e. $\vec{B} \parallel \vec{v}$

then eqⁿ ① becomes $F = 0$ i.e. minimum force.

Case II:

When $\theta = 90°$ i.e. $\vec{B} \perp \vec{v}$ then

eqⁿ ① becomes $F = Bqv$ i.e. maximum force.

Force on a current carrying straight conductor in uniform m.f.

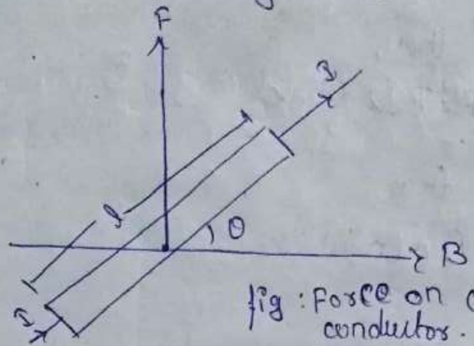


fig : Force on current carrying straight conductor.

let us consider a straight conductor of length 'l' carrying current 'I' in given direction is placed in the umf of intensity $\vec{B}$ by making angle 'θ' with conductor as shown in fig above.

If 'N' be the total number of electrons in the conductor then total charge flowing in time 't' sec through conductor is,

$$q = N.e.$$

and drift velocity of electron,

$$v = \frac{I}{enA}$$

Hence, force experienced by given electrons;

$$F = Bqv\sin\theta$$

$$= B.\,Ne.\frac{I}{enA}.\sin\theta \quad \left[n = \text{no. of electrons per unit volume}\right]$$

$$= B.\,N.\frac{I}{\frac{N}{A\times l}.A}.\sin\theta$$

$$\boxed{F = BIl\sin\theta} \quad \text{----} (1)$$

In vector form,

$$\boxed{\vec{F} = I(\vec{B}\times\vec{l})} \quad \text{----} (2)$$

Case - I:

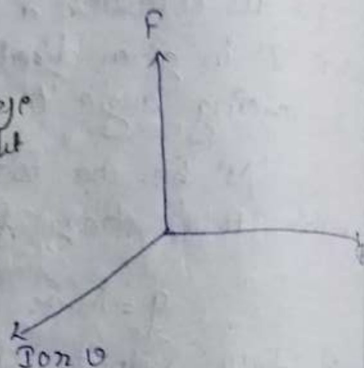
If $\theta = 90^\circ$, i.e. $\vec{B} \perp \vec{I}$ then,

Eq^n(1) becomes, $F = BIl$ (max)

If $\theta = 0^\circ$, i.e. $\vec{B} \parallel \vec{I}$ then,

Eq^n(11) becomes, $F = 0$ (min).

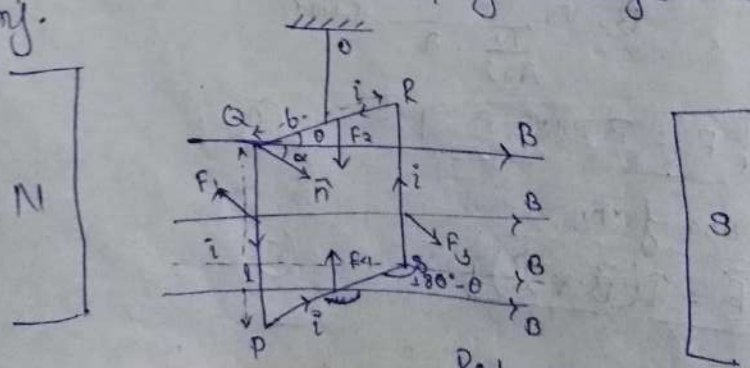
Fleming's Left Hand Rule.



(Fleming's left hand rule)

This rule is used to find the direction of force on a moving charge in umf or force on a current straight conductor in umf. According to this rule, when middle finger, forefinger and thumb are streched mutually ⊥r such that middle finger pointed the direction of velocity (v) of charge or current in the conductor and forefinger pointed the direction of mf. then, thumb will point the direction of force as shown in figure.

Torque on a current carrying Rectangular coil (loops) umf.



Rectangular coil.

let us consider a rectangular coil PQRS, having length PQ = RS = l and breadth QR = SP = b consists of n no. of turns each having area 'A' = l×b carrying current in clockwise direction. Coil is suspended from mid point of QR by a string & placed in the uniform magnetic field of intensity 'B' by ~~magnetic~~ making angle 'θ' with plane of coil as shown in figure above. Here, uniform m.f. is provided by two ~~pea~~ pole piece magnet N and S.

Now, according to Fleming's left hand rule force acting on the respective four side of coil is given by,

$F_1 = BIl$ inward

$F_2 = BIb\sin\theta$ downward

$F_3 = BIl$ outward

$F_4 = BIb\sin\theta\ (180-\theta)$
$= BIb\sin\theta$ upward.

Here, F_2 & F_4 are equal & opposite but acting along same line. So, they ~~call~~ cancelled out.

Again,
F_1 & F_3 are equal and opposite but acting along different line separated by distance,

$r = b\cos\theta$

Hence, F_1 & F_3 form a couple. Hence, torque due to this couple is given by,

τ = one of force × ⊥r distance bet^n^ forces

$\tau = BIl \times b\cos\theta$.

Put $l \times b = A$; Area of coil.

$$\tau = BIA\cos\theta \quad \text{--- (1)}$$

If 'α' be the angle made by m.f. $\vec{B}$ with normal to the plane of coil then ($\theta = 90° - \alpha$)

Eq^n (1) becomes,

$$\tau = BIA\sin\alpha \quad \text{--- (2)}$$

For 'N' no. of turns

$$\left.\begin{aligned} \tau &= BINA\cos\theta \\ \text{or, } \tau &= BINA\sin\alpha \end{aligned}\right\} \text{--- (3)}$$

<u>Case-I:</u>

If $\theta = 0°$ i.e. m.f. is parallel to plane of coil or m.f. is $\perp$r to normal to plane of coil then

$\boxed{\tau = BINA}$ maximum.

<u>Case-II:</u>

If $\theta = 90°$ or $\alpha = 0°$ i.e. m.f. is $\perp$r to plane of coil or m.f. is parallel to normal to plane of coil then $\boxed{\tau = 0}$ minimum torque.

Magnetic Dipole Moment :-

When current is flowing through a loop of conductor through m.f. $\vec{B}$ is developed forming a dipole (N-S) as shown in fig. above. Hence, magnetic diple moment is defined as the product of current and area of loop. It is denoted by "μ".

and given by, $\mu = I.A$

$$\Rightarrow I = \frac{\mu}{A} \quad ---- (1)$$

In case of current carrying rectangular loop

$$\text{Torque } (\tau) = BIA\sin\alpha \quad ---- (2)$$

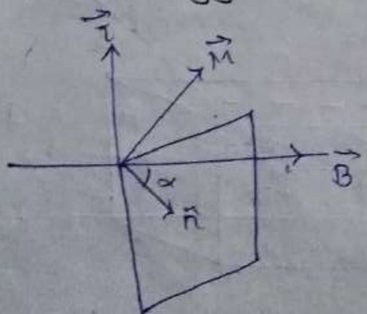
where "α" is angle made by normal to plane of coil with m.f. $\vec{B}$.

Putting value of "I" from eq^n (1) we get

$$\tau = \mu B \sin\alpha$$

In vector form $\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$ --- (3)

Eq^n(3) shows that torque is $\perp^{ar}$ to the plane containing $\vec{\mu}$ & $\vec{B}$ as shown in figure below.



Now, small amount of workdone against the torque "τ" when coil is turn by small angle dα is given by

$$dW = \tau . d\theta = \mu B \sin\alpha$$

Total workdone $W = \int_{90^\circ}^{\theta} dW$

$$\text{or, } W = \mu B \int_{90^\circ}^{\theta} \sin\alpha \, d\alpha$$

$$\text{or, } W = \mu B \left[-\cos\alpha\right]_{90^\circ}^{\theta}$$

$$\text{or, } W = \mu B \left[-\cos\theta + \cos 90^\circ\right]$$

$$\text{or, } W = -\mu B \left[\cos\theta - \cos 90^\circ\right]$$

$$\text{or, } W = -\mu B \cos\theta$$

$$\boxed{\therefore W = -\vec{\mu} \cdot \vec{B}}$$

V.V. Imp.

Hall Effect:

When current is flowing in the conductor along the direction ⊥r to applied magnetic field, then electric field is automatically developed in the conductor along the direction ⊥r to both mag. field and current. This phenomenon is known as hall effect and corresponding electric field is known as hall feild.

~~Explanati~~

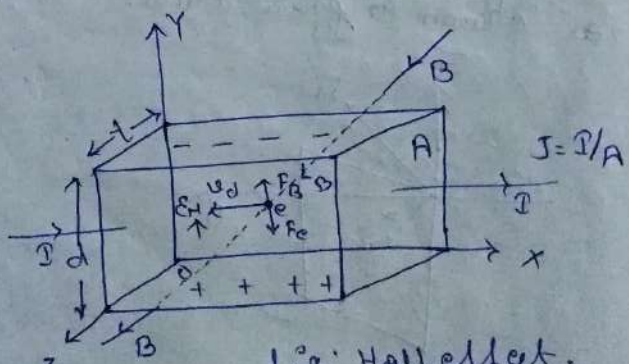


fig: Hall effect.

Explanation:

Let us consider, a conductor in the form of rectangular strip of thickness 't' having cross sectional area 'A'. If current 'I' is flowing in the conductor along 'x-axis, then current density $(J = I/A)$ is also along x-axis.

So that, drift velocity of electron,

$$v_d = \frac{J}{ne} \quad \text{----} (1)$$

Now, m.f. of intensity B is applied in the conductor along z-axis. Hence, magnetic force experienced by electron $\boxed{F_B = Bev_d}$ acting upward (+ve Y-axis) due to this force, electrons are collected at the top of conductor resulting -ve charge. According to conservation of charge, +ve charge is developed at the bottom of conductor.

As a result, ~~of~~ electric field "E_H" (Hall field) is developed along the y-axis (+ve to -ve). Due to this electric field electron experienced downward electric force,

$$F_e = -eE_H \text{ (-ve y-axis)}.$$

At equilibrium state,

$$F_e = F_B$$

$$\text{or, } e E_H = Bev_d$$

$$\therefore E_H = B.v_d$$

Put $v_d = \frac{J}{ne}$

$$\boxed{E_H = \frac{-JB}{ne}}$$

$\frac{1}{ne}$ = constant for given conductor known as Hall coefficient (R_H)

$$E_H = -R_H \cdot JB$$

$$\Rightarrow \boxed{R_H = -\frac{E_H}{JB}}$$

If "d" be the width of conductor, then, ~~Hall~~

Hall voltage, $V_H = E_H \cdot d$

$$= \frac{-JB}{ne} \cdot d$$

Put $J = I/A = \frac{I}{t \times d}$

$$\Rightarrow \boxed{V_H = -\frac{BI}{net}}$$

$-1/ne$ = constant = R_H

$$R_H = E_H/JB.$$

Mobility (μ):

When electric field is applied in the conductor, then electrons move in the direction opposite to applied electric field with an average velocity known as drift velocity. Hence, mobility of electron is defined as the drift velocity per unit applied electric field. It is denoted by μ and given by.

$$\mu = \frac{V_d}{E} \quad \text{-----} (1)$$

We know,

$$J = ne\,v_d$$

$$\Rightarrow v_d = \frac{J}{ne}$$

and $\boxed{J = \sigma E}$

where "σ" is conductivity.

$$v_d = \frac{\sigma E}{ne}$$

Now, eqn(1) becomes;

$$\mu = \frac{\sigma \cdot E}{ne \cdot E}$$

$$\boxed{\mu = \frac{\sigma}{ne}}$$

Put $1/ne = R_H$

$$\boxed{\therefore \mu = R_H \cdot \sigma}$$

Put $\sigma = 1/\rho$;

where 'ρ' is resistivity.

$$\boxed{\mu = R_H/\rho}$$

Hall resistance (R_H):

If 'V_H' be the Hall voltage & 'I' be the current flowing in the conductor than hall resistance 'R' is given by

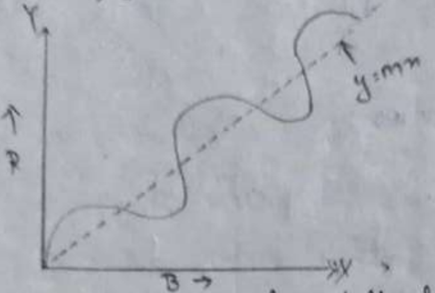
$$\cancel{R_H =} R = \frac{V_H}{I}$$

Put $V_H = \frac{BI}{net}$

$$R = \left(\frac{1}{net}\right) \cdot B$$

which is form of $y = mx$.

Hence, graph of 'B' versus R is straight line passing through origin as shown in fig below by ~~down~~ dotted line.



But, experimentally it is found that graph is non linear as shown by solid line known as quantum Hall effect.

Application of Hall Effect.

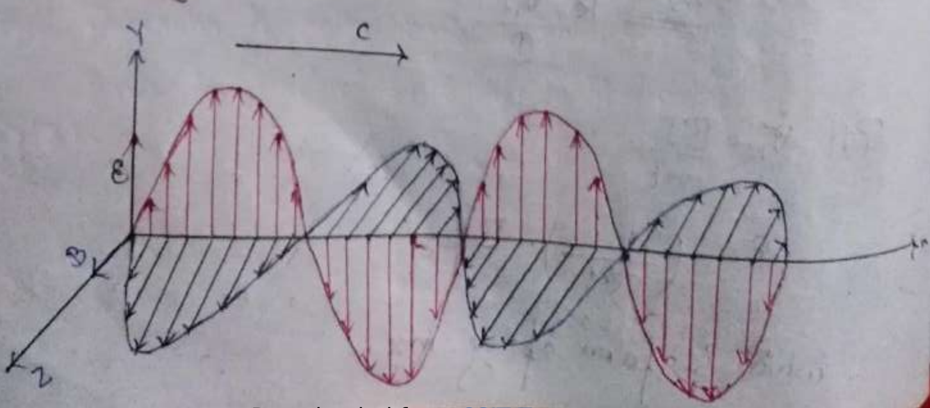
1. It is used to find the specimen/material as conductor semiconductor/insulator.
ii. It is used to find the sign of charge carrier.
iii. It is used to find the no. of charge carrier per unit volume (n).

i.e. $R_H = 1/ne$

$\Rightarrow n = \frac{1}{R_H e}$.

iv. It is used to find the mobility of electron.

Electromagnetic Wave.



When a charge is accelerated, then electric field is developed. Due to this electric field, magnetic field is induced perpendicular to the electric field. Due to this magnetic field, electric field again induced. Continuing this process, a wave is created and travel in the direction ⊥r to both electric field and magnetic field known as direction of propagation of wave. Such wave is known as ~~direction~~ electromagnetic wave. Here electric field and magnetic field are varies sinusiodally with time given by eqn.

$$\vec{E} = \vec{E_0} \sin(kx - \omega t)$$

$$\text{and } \vec{B} = \vec{B_0} \sin(kx - \omega t)$$

where $\vec{E_0}$ and $\vec{B_0}$ are maximum value of $\vec{E}$ and $\vec{B}$ respectively. 'ω' is angular frequency which is same as that of accelerating charge. "k" is wave number.

Here, the $\vec{E} \times \vec{B}$ gives the direction of propagation of wave.

The electromagnetic wave having different value of frequency is known as electromagnetic spectrum.

1. γ-ray → $\lambda = 10^{-13}$ m – 10^{-10} m
ii. X-ray → $\lambda = 10^{-11}$ m – 10^{-8} m
iii. UV ray → $\lambda = 10^{-8}$ m – 4×10^{-7} m
iv. visible lights → $\lambda = 4 \times 10^{-7}$ m – 7.8×10^{-7} m
v. Infrared ray → $\lambda = 7.8 \times 10^{-7}$ m to 10^{-3} m
vi. Micro wave → $\lambda = 10^{-3}$ m to 0.01 m
vii. Radiowave → $\lambda = 1$ m to 10^{5} m

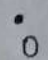
Unit: 5

Solid State Physics

Crystal Structure

A solid in three dimensional, periodic array of ions, atoms or molecules is called crystal. The periodic arrangement of atoms in crystal is called crystal lattice. The periodic arrangement of mathematical point is called surface lattice or lattice point as shown in fig(1) below:

fig(1) lattice point / space lattice



fig(2): basis

* Basis + lattice = crystal.

The groups of atoms or ions identical in composition, arrangement and orientation is called basis as shown in fig (2).

Basis is attached to every lattice point as shown in fig(3).

fig (3) : Crystal structure

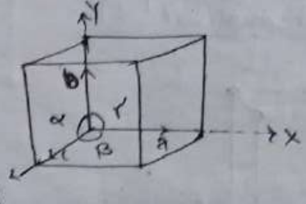
Bravais Lattice

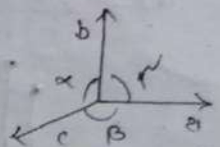
There are following fourteen types of Bravais lattice which is used to study the crystal structure.

1) Cubic Structure

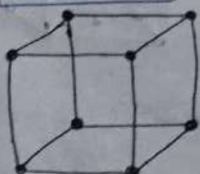
↳ $a = b = c$

↳ $\alpha = \beta = \gamma$

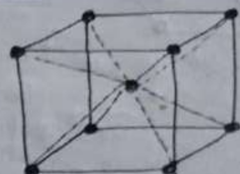




① Simple cubic (P)



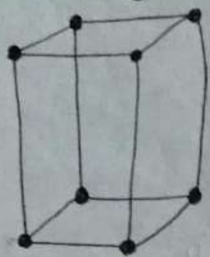
② Body centered cubic (I)



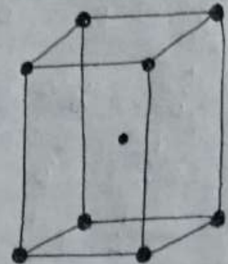
③ Face-centered cubic (F)

ii) **Tetragonal** : $a = b \neq c$ & $\alpha = \beta = \gamma = 90^\circ$

4) Simple tetragonal (P)



5) Body-centered tetragonal (I)

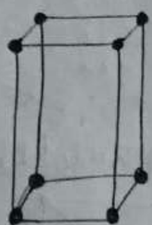


iii) **Orthorhombic**

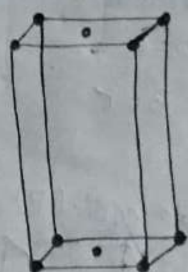
$a \neq b \neq c$

$\alpha = \beta = 90^\circ$

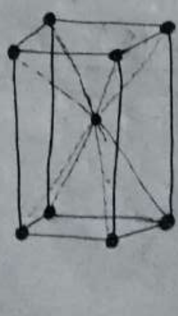
6) Simple orthorhombic (P)



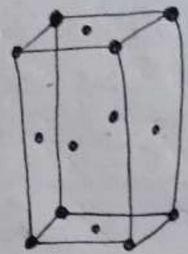
7) Base-centered orthorombic (C)



8) Body-centered orthorhombic



9) Face - centered orthorhombic

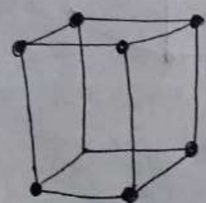


iv) Monoclinic

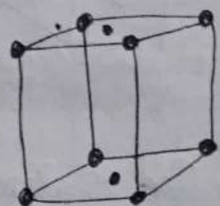
$\alpha = \gamma = 90^\circ \neq \beta$

$a \neq b \neq c$

10) Simple monoclinic



11) Base centered monoclinic

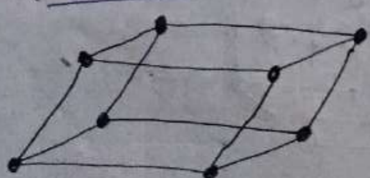


v) Triclinic

$\alpha \neq \beta \neq \gamma$

$a \neq b \neq c$

12) Triclinic

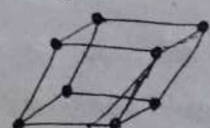


vi) Trigonal

~~α =~~ $\alpha = \beta = \gamma = 90^\circ < 120^\circ$

$a = b \neq c$

13) Trigonal



13) Hexagonal

$\alpha = \beta = 90^\circ$

$\gamma = 120^\circ$

$a = b \neq c$

14) Hexagonal

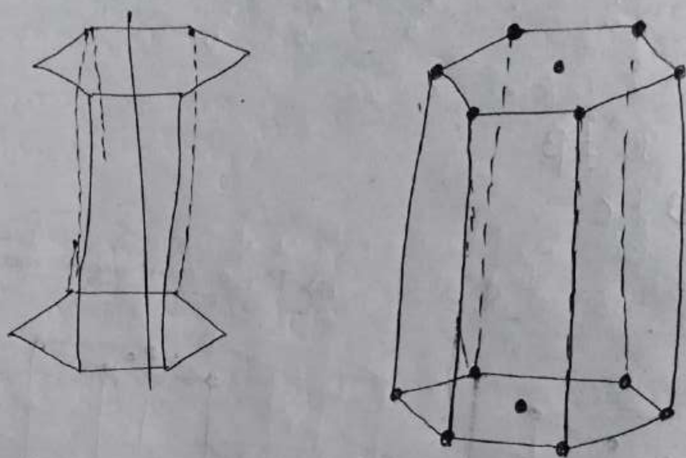
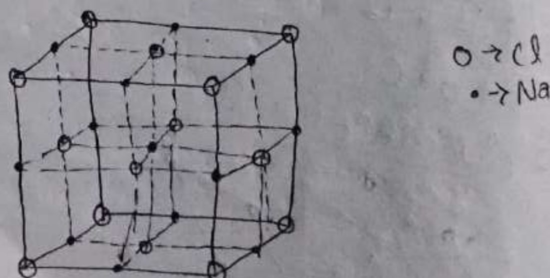


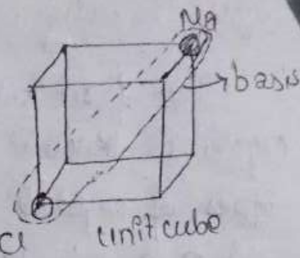
fig: The 14 Bravais space lattices.

NaCl Structure



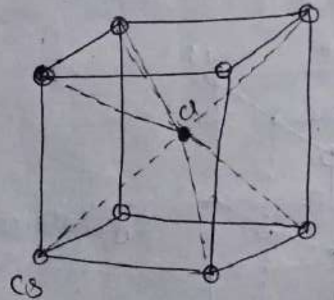
Face centered Crystal (F)
(FCC)

It is a FCC lattice.
Basis consist of Na atom and Cl atom separated by 1/2 of body diagonal of unit cube other KBR, AgBr, MnO etc.

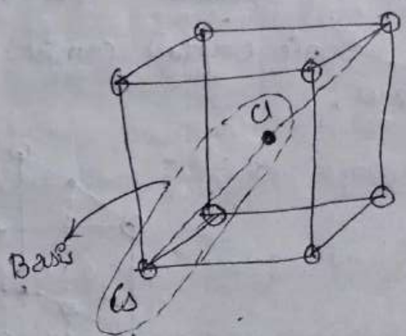


CsCl Structure:

This is a simple cubic Brava is lattice as shown in fig (1). The basis consists of a Cl atom at the corner and a Cs atom separated by one-half the body diagonal, fig (11). Other materials having this structure including AgMg, AlNi, CuZn (brass), & Belu.



(1) cubic crystal structure of cesium crystal (CsCl)



(11) Basis of the CsCl crystal structure.

Crystal Bonding

The force which holds ions, molecules or atoms together in crystal is known as crystal bonding. There are following five types of crystal bonding;

1. Ionic bonding
2. Covalent bonding.
3. Vanderwaal bonding.
4. Hydrogen bonding
5. Metallic bonding.

Ionic bonding.

$$NaCl \rightarrow Na^+ + Cl^-$$

Ionic crystals consist of +ve and -ve ions as shown in fig below,

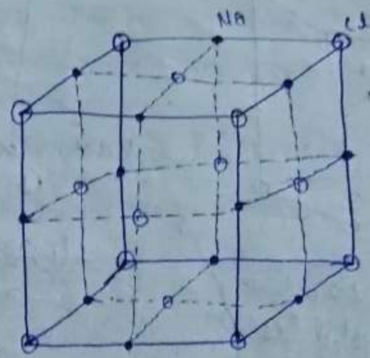


fig: NaCl crystal.

This is a result of losing of electron from one atom to another atom. Atoms in ionic crystal are bounded by electro-static force.

Eg

Eg: Formation of NaCl crystal

$Na + 5.14 eV \longrightarrow Na^+ + e^-$ ---- (i)
(absorbed)

$Cl + e^- \longrightarrow Cl^- + 3.62 eV$ ---- (ii)
(released)

If we bring Na^+ and Cl^- together until the separation between them $r = 2.51 A^\circ = 2.51 \times 10^{-10}$ m ie equilibrium separation the columb attraction,

$$P.E., E_p = \frac{-e^2}{4\pi\epsilon_0 r}$$

$$E_p = \frac{-9.9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(2.51 \times 10^{-10})}$$

$$E_p = -5.73 \times 1.6 \times 10^{-19} J = -5.73 eV$$

Hence, Net energy released

$$E = 5.14 - (3.62 + 5.73)$$

$$= -4.2 eV$$

which is also energy required to break NaCl crystal known as bounded energy of NaCl.

Hence, any ionic crystal B.E is sum of an attraction and repulsive force

Thus, B.E. can be written as,

$$E = -\frac{\alpha e^2}{4\pi\epsilon_0 r}.$$

where, "α" is known as Medelung's constant.

For, FCC, NaCl, $\alpha = 1.7476$.

VV Imp.
Free electron theory of metal.

In metals, electrons are loosely bound their atoms. So they are free to move just like gas molecule known as electron gas. The +ve ions at the lattice produce attractive potential energy so that electrons are confined with it's P.E. known as potential well.

There are two types of free electron theory in metal,

1. Classical free electron theory or model (CFEM)
2. Quantum mechanical free electron theory or model (QFEM)

Classical Free Electron Theory

There are following four basic assumptions in this model.

1) Metal is composed of an array of ions with valence electrons that are free to move with only recozition that they are remains confined with in a deep boundaries of metal. And valence electrons are responsible for condition.

2) Free electrons obeys classical Maxwell-Boltzmann statistics.

3) Electrons are moving average ~~roun a~~ random velocity 'v' given by

$$\frac{1}{2} m v^2 = \frac{3}{2} KT.$$

where, $m = 9.1 \times 10^{-31}$ kg] mass of electron.

$K = 1.38 \times 10^{-28}\ J\ K^{-1}$ is Boltzmann constant.

'T' is absolute temperature of electron gas.

4) When electric field is applied in the metal, the electrons are move with average drift velocity 'v_d' in the direction opposite

to applied electric field.

Derivation of ohm's law from CFEM.

(CFEM → Classical Free Electron Theory).

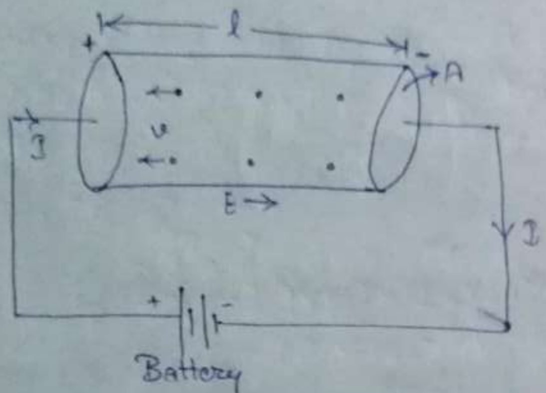


fig: Conduction of electron.

Derivation of Thermal Conductivity (η) From CFEM. $\left[\eta = \frac{1}{2} n v_{rms}^2 \tau K\right]$

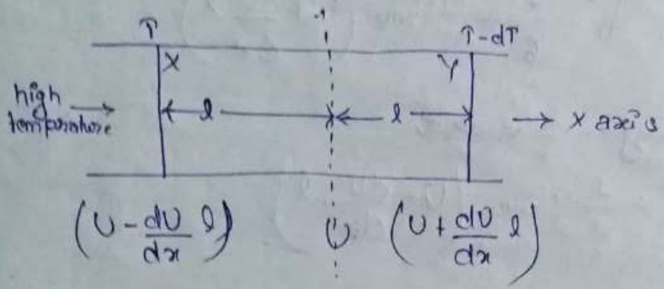


fig: flow of heat.

let us consider a part XY of conductor having cross-sectional area (A) and temperature gradient ~~of~~ $\left(\frac{dT}{dx}\right)$.

In the x-axis, if J_n be the heat current density then,

(i) $J_n \propto A$

& (ii) $J_n \propto \frac{dT}{dx}$

Combining above eq^n,

$$J_n \propto A \frac{dT}{dx}$$

$$\boxed{J_n = \eta A \frac{dT}{dx}} \quad \text{------ (1)}$$

where η is proportionality constant known as thermal conductivity.

From eq^n(1),

Heat current density $J_n = \frac{J_n}{A} = \eta \frac{dT}{dx}$ ------ (2).

If the region XY is filled with electron than gas then, current density along the x-axis is,

$$J_h^+ = \frac{1}{6} n v_{rms} \left(U + \frac{dU}{dx} l \right)$$

and along -ve x-axis,

$$J_h^- = \frac{1}{2} n v_{rms} \left(U - \frac{dU}{dx} l \right)$$

factor $\frac{1}{6}$ for six directions $\pm x, \pm y, \pm z$.

$n v_{rms}$ is no. of electrons crossing per unit area per time through any plane.

'v_{rms}' is rms velocity of electron gas.

'$\frac{dU}{dx}$' is rate of flow of heat with distance.

$\therefore$ Total heat current density

$$J_h = J_h^+ - J_h^-$$

$$J_h = \frac{1}{6} n v_{rms} \cdot 2 \frac{dU}{dx} \cdot l \quad \text{--- (3)}$$

We know, sp. heat capacity of electron gas at constant volume

$$C_v = \frac{dU}{dT} \cdot N_A$$

$$\Rightarrow dU = \frac{C_v dT}{N_A}$$

∴ eqn(3) becomes,

$$J_n = \frac{1}{3N_A} n v_{rms} l C_v \frac{dT}{dx} \quad ----(4)$$

Comparing eqn (2) & (4),

$$\eta = \frac{1}{3N_A} n v_{rms} \cdot l \cdot C_v$$

Put $C_v = \frac{3}{2}R$ & $l = v_{rms} \cdot \tau$] $\tau \rightarrow$ relaxation time.

$$\eta = \frac{1}{3N_A} n \cdot v_{rms}^2 \cdot \tau \cdot \frac{3}{2} R$$

Put $\frac{R}{N_A} = k$. Boltzmann constant

$$\boxed{\eta = \frac{1}{2} n v_{rms}^2 \tau k}$$

which is the required expression.

Weidemann Frank law.

The ratio of electrical conductivity ~~and therm~~ (σ) and thermal conductivity (η) is always ~~constant~~ directly propor- ~~Hence~~, tional to absolute temperature 'T' of electron gas.

$$\frac{\eta}{\sigma}$$

According to this, the ratio of thermal conductivity 'η' and electrical conductivity 'σ' is always directly proportional to absolute temperature 'T' of electron gas.

$$\frac{\eta}{\sigma} \propto T.$$

Proof:

Here, $$\frac{\eta}{\sigma} = \frac{\frac{1}{2} n v_{rms}^2 \tau K}{\frac{n e^2 \tau}{m}}$$

$$\frac{\eta}{\sigma} = \frac{\frac{1}{2} m v_{rms}^2 K}{e^2}$$

Put $\frac{1}{2} m v_{rms}^2 = \frac{3}{2} K T$

$$\frac{\eta}{\sigma} = \frac{3}{2}\left(\frac{K}{e}\right)^2 T$$

$$\boxed{\frac{\eta}{\sigma} = L \cdot T}$$

where $L = \frac{3}{2}\left(\frac{K}{e}\right)^2$ is a constant. also known as lorent number whose value is $\frac{3}{2}\left(\frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}}\right)^2$

$= 1.12 \times 10^{-8}\, W\Omega K^{-2}$

2. Quantum mechanical free electron model (QFEM)

Sommerfield modified free electron model in two way

1. The electron must be treated quantum mechanically.
2. The electron must obeys Paulli's exclusion principle i.e. no two electrons can have same set of quantum number.

for this he made the following three assumptions

1. The valence electron in metals are free to move.
2. Interaction between electron and lattice is neglected.
3. Interaction between electron is neglected.

Three Dimensional Fermi Energy.