Tribhuwan University
Institute of Science and Technology
2065

Bachelor Level / First Semester / Science | Full marks: 80
Computer Science and Information Technology(MTH112) | Pass marks: 32
((TU CSIT) Mathematics I (Calculus)) | Time: 3 hours
Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions.

Group A

1. Verify Rolle's theorem for the function $f(x) = \frac{x^3}{\text{[illegible]}} - 3x$ on the interval [-3, 3].

2. Obtain the area between two curves y = sec2x and y = sin x from x = 0 to x = π/4.

3. Test the convergence of p – series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for p > 1.

4. Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.

5. Find a vector perpendicular to the plane of P(1, -1, 0), C(2, 1, -1) and R(-1, 1, 2).

6. Find the area enclosed by the curve $r^2 = 4\cos 2\theta$.

7. Obtain the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4, -5) if $f(x,y) = x^2 + 3xy + y - 1$.

8. Using partial derivatives $\frac{dy}{dx}$, find if $x^2 + \cos y - y^2 = 0$.

9. Find the partial differential equation of the function $(x - a)^2 + (y - b)^2 + z^2 = c^2$.

10. Solve the partial differential equation $x^2p + q = z^2$.

Group B

11. State and prove the mean value theorem for a differential function.

12. Find the length of the Asteroid $x = \cos^3 t$, $y = \sin^3 t$ for $0 \leq t \geq 2\pi$.

13. Define a curvature of a curve. Prove that the curvature of a circle of radius a is 1/a.

14. What is meant by direction derivative in the plain? Obtain the derivative of the function $f(x,y) = x^2 + xy$ at P(1, 2) in the direction of the unit vector $v = \left(\frac{1}{\sqrt{2}}\right) i + \left(\frac{1}{\sqrt{2}}\right) j$

15. Find the center of mass of a solid of constant density δ, bounded below by the disk: $x^2 + y^2 = 4$ in the plane z = 0 and above by the paraboid $z = 4 - x^2 - y^2$.

16. Graph the function f(x) = $-x^3 + 12x + 5$ for $-3 \leq x \leq 3$.

Group C

17. Define Taylor's polynomial of order n. Obtain Taylor's polynomial and Taylor's series generated by the function f(x) = e^x at x = 0.

18. Obtain the centroid and the region in the first quadrant that is bounded above by the line y = x and below by the parabola y = x^2.

19. Find the maximum and the minimum values of f(x, y) = $2xy - 2y^2 - 5x^2 + 4x - 4$. Also find the saddle point if it exists.

OR

Evaluate the integral $\int_0^{\sqrt{2}} \int_0^{3y} \int^{6-x^2-y^2} \mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$

20. What do you mean by d' Alembert's solution of the one-dimensional wave equation? Derive it.

OR

Find the particular integral of the equation ($D^2 - D^1$)z =$2y-x^2$ where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$