Tribhuwan University
Institute of Science and Technology
2067

Bachelor Level / First Semester / Science — Full marks: 80
Computer Science and Information Technology(MTH112) — Pass marks: 32
((TU CSIT) Mathematics I (Calculus)) — Time: 3 hours
Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions.

Group A (10×2=20)

1. Define a relation and a function from a set into another set. Give suitable example.

2. Show that the series $\sum^{\infty} \frac{1}{n^2}$ **converges by using integral test.**

3. Investigate the convergence of the series $\sum^{\infty} \frac{2^n+5}{3^x}$

4. Find the foci, vertices, center of the ellipse $\frac{x^2}{} + \frac{y^2}{} = 1$

5. Find the equation for the plane through (-3,0,7) perpendicular to

6. Define cylindrical coordinates (r, v, z). Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical coordinates.

7. Calculate $\iint f(x, y)\,dA$ **for $f(x,y) = 1 - 6x^2y$, $R : 0 \leq x \leq 2, -1 \leq y \leq 1$.**

8. Define Jacobian determinant for x = g(u, v, w), y = h(u, v, w), z = k(u, v, w).

9. What do you mean by local extreme points of f(x,y)? Illustrate the concept by graphs.

10. Define partial differential equations of the first index with suitable examples.

Group B (5×4=20)

11. State the mean value theorem for a differentiable function and verify it for the function

f(x) = $\sqrt{1 - x^2}$ **on the interval [-1,1].**

12. Find the Taylor series and Taylor polynomials generated by the function f(x) = cos x at x = 0.

13. Find the length of the cardioid r = 1 – cosθ.

14. Define the partial derivative of f(x,y) at a point (x_0, y_0) with respect to all variables. Find the derivative of f(x,y) = xe^y = cos(x, y) at the point (2, 0) in the direction of A = 3i – 4j.

15. Find a general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z.$

Group C (5×8=40)

16. Find the area of the region in the first quadrant that is bounded above by y = √x and below by the x-axis and the line y = x – 2.

OR

Investigate the convergence of the integrals

(a) $\int_{1}^{0} \frac{1}{1-x} \mathrm{d}x$ (b) $\int_{0}^{3} \frac{dx}{x-1^{2/3}}$

17. Calculate the curvature and torsion for the helix r(t) = (a cos t)i + (a sin t)j + btk,a,b ≥ 0, $a^2 + b^2 \neq 0$.

18. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

19. Find the absolute maximum and minimum values of $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by lines x = 0, y = 0 and x + y =9.

OR

Find the points on the curve $xy^2 = 54$ nearest to the origin. How are the Lagrange multipliers defined?

20. Derive D' Alembert's solution satisfying the initials conditions of the one-dimensional wave equation.