Tribhuwan University
Institute of Science and Technology
2068

Bachelor Level / First Semester / Science	Full marks: 80
Computer Science and Information Technology(MTH112)	Pass marks: 32
((TU CSIT) Mathematics I (Calculus))	Time: 3 hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Attempt all questions.

Group A (10×2=20)

1. Define one-to-one and onto functions with suitable examples.

2. Show by integral test that the series $\sum^{\infty} \frac{1}{x^p}$ **converges if p>1.**

3. Test the convergence of the series $\sum^{\infty} (-1)^{x+1} \frac{1}{x^2}$

4. Find the focus and the directrix of the parabola $y^2 = 10x$.

5. Find the point where the line X = 8/3 + 2t, y = -2t, z = 1 + t intersects the plane 3x + 2y + 6z = 6.

6. Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.

7. Find the area of the region R bounded by y = x and $y = x^2$ in the first quadrant by using double integrals.

8. Define Jacobian determinant for X = g(u, v, w) ,y = h(u, v, w), z = k(u, v, w).

9. Find the extreme values of $f(x,y) = x^2 + y^2$.

10.Define partial differential equations of the second order with suitable examples.

Group B (5×4=20)

11. State Rolle's Theorem for a differential function. Support with examples that the hypothesis of theorem are essential to hold the theorem.

12. Test if the following series converges

(a) $\sum_{n=1}^{\infty} \frac{x^2}{2^x}$ (b) $\sum_{n=1}^{\infty} \frac{2^x}{x^2}$

13. Obtain the polar equations for circles through the origin centered on the x and y axis and radius a.

14. Show that the function $f(x) = \begin{cases} \frac{2xy}{x^2+y^2}, (x, y) \neq (0,0) \\ 0 \quad (x, y) = 0 \end{cases}$ **is continuous at every point except the origin.**

15. Find the solution of the equation $\frac{\partial^2 y}{} - \frac{\partial^2 z}{} = x - y$

Group C (5×8=40)

16. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

OR

Evaluate the integrals

(a) $\int_0^3 \frac{dx}{(x-1)^{2/3}}$ (b) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

17. Define a curvature of a space curve. Find the curvature for the helix $r(t) = (a \cos t)i + (a \sin t)j + btk (a,b \geq 0, a^2 + b^2 \neq 0)$.

18.Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

19. Find the maximum and minimum values of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

OR

State the conditions of second derivative test for local extreme values. Find the local extreme values of the function $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$.

20. Define one-dimensional wave equation and one-dimensional heat equations with initial conditions. Derive solution of any of them.