

Tribhuvan University
Institute of Science and Technology
2069

Bachelor Level / First Semester / Science

Computer Science and Information Technology(MTH112)

((TU CSIT) Mathematics I (Calculus))

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Full marks: 80

Pass marks: 32

Time: 3 hours

Attempt all questions.

Group A (10×2=20)

1. Verify the mean value theorem for the function $f(x) = \sqrt{x}(x-1)$ in the interval $[0, 1]$.

2. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$ for $0 \leq x \leq 1$.

3. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. By comparison test.

4. Obtain the semi-major axis, semi-minor axis, foci, vertices $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

5. Find the angle between the vectors $2i+j+k$ and $-4i+3j+k$.

6. Obtain the area of the region R bounded by $y=x$ and $y=x^2$ in the first quadrant.

7. Show that the function $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ is continuous at every point in the plane except the origin.

8. Using partial derivatives, find $\frac{dy}{dx}$ if $2xy + \tan y - 4y^2 = 0$.

9. Verify that the partial differential equation $\frac{d^2 Z}{dx^2} + \frac{d^2 Z}{dy^2} = \frac{2Z}{x^2 + y^2}$ is satisfied by $Z = \frac{1}{x^2 + y^2}$.

10. Find the general solution of the equation $x^2 \frac{dz}{dx} + y^2 \frac{dz}{dy} = (x+y)z$.

Group B (5×4=20)

11. State and prove mean value theorem for definite integral.

12. Find the area of the region that lies in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

13. What do you mean by principle unit normal vector? Find unit tangent vector and principle unit vector for the circular motion $\vec{r}(t) = (\cos 2t)i + (\sin 2t)j$.

14. Define partial derivative of a function $f(x,y)$ with respect to x at the point (x_0, y_0) . State Euler's theorem, verify if it for the function $f(x,$

$$y) = x^2 + 5xy + \sin x + 7e^x, \quad x = \left(\frac{y}{2}\right) + 1$$

15. Find the particular integral of the equation $\frac{d^2z}{dx^2} - \frac{dz}{dx} = 2x - x^2$

Group C (5×8=40)

16. Graph the function $y = \frac{1}{x^2} - \frac{2}{x^3}$

17. What is mean by maclaurin series? Obtain the maclaurin series for the function $f(x) = e^{-x}$

18. Evaluate the double integral $\int_0^4 \int_{\frac{x-y}{2}+1}^{\frac{x-y}{2}+1} \frac{2x-y}{x} dx dy$ by applying the transformation $u = \frac{2x-y}{x}, v = \frac{y}{x}$ and integrating over an appropriate region in the uv -plane.

19. Define maximum and minimum of a function at a point. Find the local maximum and local minimum of the function $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$.

20. Find the solution of the equation $\frac{d^2z}{dx^2} - \frac{dz}{dx} = x - y$

Or

Find the particular integral of the equation $(D^2 - D)z = 2y - x^2$ Where $D = \frac{d}{dx}, D' = \frac{d}{dy}$